The calculation of Indicator cards in ‘Perform.exe’

1. The program ‘Perform.exe’ calculates an indicator diagram on the basis that the steam behaves essentially as a gas – that is, there is no condensation. It does, of course, take account of the pressure drop between steam chest and cylinder, and therefore predicts the resulting ‘wiredrawing’. In addition it calculates the flow rate of steam through the inlet ports, and is thus able to present not only the IHP but also the Steam Rate. The following notes are intended as an amplification of the description of the program given in the note “Predicting Locomotive Performance” (SLS Journal Vol.75, No.797, 1999)

2. Valve events are calculated from a valvegear model that neglects angularity effects in the (Walschaerts) gear but includes the more important angularity of the connecting rod. This latter effect is evident in the program output in that the front and back cards are not identical. The model has been checked against an exact valvegear program (Walschaerts.exe) and is found to be in good agreement in the case of BR Class 7 gear. It is likely to prove adequate for all well designed Walschaerts gears, but a version of Perform.exe which uses an exact calculation is on the stocks so that a further check may be made. Since the model provides the valve motion around the cycle, the port opening corresponding to any crank angle is known. When running the program the cut-off is specified and the other events are calculated using the specified lap and lead; again, these events correspond well with the exact valvegear program.

3. Compressible flow equations are used to provide a relationship between pressure drop across the ports and the mass flow of steam through them. In many situations the flow reaches sonic velocity in one or both inlet and exhaust ports, so it is important to recognise choked flow. ‘Polytropic’ expansion and compression are assumed, and the exponents are inputs in the program. A constant coefficient of discharge of the ports is applied, and this can also be varied in the program.

4. The central equation to be solved (Equation 10) relates a mass balance for the cylinder (based on the flow through the valves) to the change of pressure. Initially the pressure at TDC is taken to be the steam chest pressure. The piston is allowed to move (in elements of time corresponding to 1° rotation of the crank) and a balance is then struck between the expansion of the steam and the flow of new steam into the cylinder, generated by the reduction in pressure. This process is repeated at 1° intervals around the cycle, taking into account the inlet and exhaust port openings. First time around there may be a difference between the final cylinder pressure and the assumed initial pressure at TDC, so a new initial pressure is chosen and the procedure repeated until initial and final pressures are the same; of course they are not always as high (at TDC) as the steam chest pressure, particularly if the lead is inadequate at high speed.

5. The above procedure is carried out by a numerical analysis routine for integrating differential equations – the Runge-Kutta routine. This routine chooses steps of time (in between the 1° intervals) that are small enough to ensure a specified accuracy for the process. A further iteration is required because the exhaust pressure is not known initially. The steam flow rate is used to calculate the pressure across the blast nozzle, thus giving a new exhaust pressure, and the whole process is again repeated until the exhaust pressure corresponds to the calculated pressure across the blast nozzle. In spite of the size of the calculation, which would take days if carried out by hand, a reasonably fast computer completes it in a few seconds.

6. Whilst calculating the indicator diagram, the Runge-Kutta routine integrates the flow into the cylinder during admission and also calculates the area of the diagram, thus giving the mean effective pressure. The calculation is made for each end of the cylinder, and the MEP and the steam flow are then averaged, giving single values for IHP and Steam Usage. When over-compression occurs, giving a compression loop, two alternative assumptions can be made when analysing the diagram: either the compressed steam can be allowed to flow back into the steam chest (thus

reducing the steam consumption) or it can be allowed to escape to exhaust. Both results are presented by the program. The compression loop reduces the MEP and IHP, but as yet no adjustment is made for the case when the steam is released to exhaust; this will depend on the characteristics of the relief valves. Until such a modification can be made the calculated IHP may be too low when compression loops exist. The program illustrates rather well the effects of short cut-off, when admission and compression occurs earlier and the valve opening is reduced; such effects dictate the form of the compression loop.

7. The main deficiency of the program is the neglect of condensation. Comparison with test data suggests that such effects are small with reasonable superheat and at high speed, but may be important at low speed. My current view is that the mean temperature of the cylinder block, covers and piston may be held rather close to the saturation temperature of the inlet steam, and that some condensation and subsequent evaporation probably always occurs. Heat transfer between steam and cylinder is greatly enhanced by condensation, compared with convection between superheated steam and a dry cylinder. Such heat transfer will only penetrate a few mm into the metal, thus producing a layer in which the temperature fluctuates, whilst the bulk temperature of the cylinder remains steady. The important point to recognise is that the heat of condensation is, later in the cycle, removed by evaporation of the condensate layer; there will, of course be some heat loss from the outside of the cylinder, but this is probably very small compared with the potential heat fluxes into and out of the cylinder caused by condensation and evaporation.

8. Inclusion of condensation will greatly complicate the calculations in perform.exe, and as yet no clear model for the process has been set up. The thought is that the best starting point will be the start of compression, when any condensate will probably have evaporated, and the surface temperatures of cover and piston will be at their lowest point. Condensation will then occur on these surfaces if the steam pressure exceeds the saturation pressure corresponding to the (unknown) surface temperature. It will be necessary to follow the subsequent condensation and evaporation around the cycle, and to assess the effect on surface temperatures. Yet another iterative process!

9. Finally, a comment on the ‘knobs’ provided to adjust the behaviour of Perform.exe. These have been kept to a minimum by basing the calculation on well established physical principles wherever possible. In two cases coefficients have been introduced where behaviour may be affected by design details that are not easily available, or where comparison can be made with experimental data. The first case is the coefficient of discharge to be applied to flow through the ports. Experimental data are not usually available, but the process is similar to flow through nozzles and orifices. A nozzle with a well rounded inlet has a coefficient very close to 1, whereas a sharp edged circular orifice has a coefficient of around 0.6. An obvious starting point for the program is 0.8. The second case is the blastpipe and the passages connecting it with the exhaust ports. The nozzle itself, if fitted with a well rounded contraction would have a coefficient approaching 1, but to allow for the small pressure drops in the exhaust passages a value of 0.9, which can be adjusted, is selected. The other adjustable parameters are the exponents of the polytropic expansion and compression. Probably in most cases of interest the steam will be superheated, in which case an exponent of 1.3 is appropriate. Wet steam has an exponent of 1.135.

10. The appendices to the paper “Predicting Locomotive Performance” are appended; these give the equations on which the program Perform.exe is based.

W B Hall, Sept, 2001.

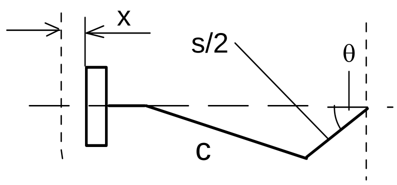
APPENDIX 1 : DETAILS OF THE ANALYSIS

Motion of Piston and Valve

Piston

The piston displacement, x and the dimensionless displacement ξ are given by:

$$\xi = x/s = c/s + 0.5 - \sqrt{(c/s)^2 - (\sin\theta/2)^2} - \cos\theta/2 \quad . . (1)$$



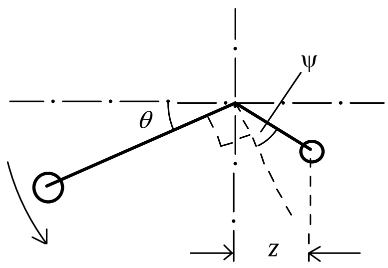
where c = length of connecting rod
s = stroke

(When determining overall performance characteristics that depend upon both strokes of the piston angularity effects are eliminated by assuming c / s to be very large.)

Valve

Assume the "equivalent eccentric" model, in which the valve motion is completely determined by the steam lap L, the lead l, and the crank angle at cut-off θ_c . The port openings are determined by these quantities together with the exhaust lap and the port width. The equivalent eccentric is, as its name implies, an eccentric with a radius r_{eq} and angle of advance ψ such that it represents the motion of the valve; as the real valve gear is "notched up" this radius and angle of advance will change. We must therefore relate r_{eq} and ψ to the independent variables L, l, and θ_c. (Usually it is the fractional cut-off ξ_{co} that is specified rather than the value of the crank angle at cut-off. However the two are related by Equation (1)

(Equation (1) is inverted using a root-finder in the computer programme to give θ as a function of ξ_{co})



The valve displacement from its mid position is

$$z = r_{eq}\sin(\theta + \psi) \quad . . . . (2)$$

At front dead centre *(θ = 0)* , the inlet port is open by an amount l, so that :

$$L + l = r_{eq}.\sin\psi$$

Also, at cut-off, when the crank angle is θ_c , the displacement of the valve from its mid-point is L, so that: $L = r_{eq}.\sin(\theta_c + \psi)$

It follows from these two conditions that :

$$\psi = \tan^{-1}\left\{ \sin\theta_c \Big/ \left[L/(L+1) - \cos\theta_c \right] \right\} \quad \text{and} \quad r_{eq} = (L+l)/\sin\psi \quad . . . . (3)$$

With these definitions, the port openings can be expressed as:

Inlet port opening , $y_i = r_{eq}.\sin(\theta + \psi) - L$ . . . . (4)

Exhaust port opening , $y_o = -r_{eq}.\sin(\theta + \psi) - L_{ex}$ . . . . (5)

The maximum opening must of course be limited to the actual width of the port.

Flow through the ports

Admission phase

During admission the pressure upstream of the valve is assumed to remain constant at the steam chest pressure. At the start of a stroke the cylinder pressure is usually fairly close to the steam chest pressure, but as the piston accelerates the cylinder pressure falls and the speed of flow of steam through the valve increases. If the valve is regarded as an orifice discharging into a vessel (the cylinder) in which the pressure is uniform, one can calculate the instantaneous speed of flow of the steam into the cylinder. As the cylinder pressure drops, so the steam speed through the valve increases; in some cases it may reach the speed of sound so that the flow becomes choked and any further decrease in cylinder pressure does not affect the speed. (What this really means is that there is no way that messages from downstream can be transmitted upstream of the orifice to tell the steam to speed up; such messages travel at the speed of sound and can therefore make no headway against the sonic velocity of the steam at the throat of the orifice). This phenomenon occurs at a particular value of the ratio of downstream to upstream pressure (the ‘critical pressure ratio’): for superheated steam the critical pressure ratio is about 0.55. The calculation must therefore take account of the fact that no further increase in steam speed is possible if the pressure ratio falls below this value. Having found the steam speed we can then use the upstream density and the port opening from Equation (4) to calculate the mass flow rate of steam into the cylinder.

Calculation of the mass flow is straightforward enough but we need to know the volume of steam admitted - corresponding to its specific volume in the cylinder. The specific volume after admission, when the steam is virtually at rest can be determined by assuming that the overall process, of discharge through the valve followed by stagnation, is one of constant enthalpy. For steam in the range of conditions of interest, constant enthalpy is quite well represented by a constant value of the product *pv*, so that if the upstream pressure and specific volume are p_o and v_o and the cylinder pressure is *p*, the specific volume of the steam in the cylinder will be $p_o v_o / p$. The flow rate through the valve is determined by the ratio of the upstream pressure to the pressure at the ‘throat’ of the valve. For subsonic flow this pressure is the same as the cylinder pressure, *p*; for sonic flow it will be a pressure (greater than *p*) determined by the upstream pressure and the critical pressure ratio. Thus we define the throat pressure, p', so that when the flow is choked it is equal to the critical throat pressure for sonic flow; otherwise it is equal to the cylinder pressure. The following equations are based on standard relationships for reversible adiabatic flow of a fluid that follows the expansion law $pv^n = constant$. (Note that whilst overall flow process is irreversible, the flow to the ‘throat’ will approach reversibility). The variable cross sectional area of the inlet port is denoted by A_i, and the coefficient of discharge by C_d.

$$\textit{Mass flow rate, } M = C_d \cdot A_i \cdot \frac{1}{v_o}\left(\frac{p'}{p_o}\right)^{\frac{1}{n}} \cdot \sqrt{2\left(\frac{n}{n-1}\right)p_o v_o\left\{1-\left(\frac{p'}{p_o}\right)^{\frac{n-1}{n}}\right\}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$\textit{Specific volume in cylinder} = v_o\, p_o \,/\, p$$

$$\textit{Volume flow rate to cylinder} = C_d \cdot A_i \cdot \sqrt{p_o v_o}\left(\frac{p_o}{p}\right)\sqrt{2\frac{n}{n-1}\left\{\left(\frac{p'}{p_o}\right)^{\frac{2}{n}}-\left(\frac{p'}{p_o}\right)^{\frac{n+1}{n}}\right\}} \quad \ldots\ldots\ldots\ldots\ldots \mathbf{(7)}$$

where:

p = cylinder pressure, p_o = steam chest pressure, p' = throat pressure,

$p' = p$ when $p > p_c$ and

$p' = p_c$ when $p < p_c$

$$\frac{p_c}{p_o} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$$ (the 'critical pressure ratio' for flow through the inlet valve)

Exhaust phase

This is as for the Admission phase, except that in this case the pressure upstream of the valve is the pressure in the cylinder, which of course is not constant. On the other hand the pressure downstream is constant. The specific volume of the steam in the cylinder is obtained by assuming that the expansion in the cylinder follows the law $pv^n = constant$. This is not strictly true because of the irreversibilities in the expansion through the inlet port; however the discrepancy introduced affects only the estimation of velocity and is not thought to be significant. Choking may occur if the exhaust pressure is less than the critical pressure for sonic flow through the exhaust ports. As with flow through the inlet valve we must distinguish between the downstream pressure and the critical pressure in determining the flow velocity. For flexibility, the calculation also allows a different expansion index m for flow through the exhaust ports; this index also applies during the compression phase. The cross sectional area of the exhaust ports is A_o

$$Mass\ \ flow\ rate\mathbf{,}\ M = C_d \,\mathbf{.}\, A_o \,\mathbf{.}\, \frac{1}{v}\left(\frac{p'}{p}\right)^{\frac{1}{m}} \sqrt{2\left(\frac{m}{m-1}\right) pv \left\{1 - \left(\frac{p'}{p}\right)^{\frac{m-1}{m}}\right\}} \qquad \mathbf{.\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ (8)}$$

And since the specific volume of steam leaving cylinder at pressure p is $M \times v$,

$$Volume\ \ flow\ rate\ of\ \ steam\ leaving\ cylinder = C_d \,.\, A_o \sqrt{p_o v_o}\left(\frac{p}{p_o}\right)^{\frac{n-1}{2n}} \sqrt{2\frac{m}{m-1}\left\{\left(\frac{p'}{p}\right)^{\frac{2}{n}} - \left(\frac{p'}{p}\right)^{\frac{m+1}{m}}\right\}} \quad ..(9)$$

..

where

p = cylinder pressure, p_e = steam chest pressure, p' = throat pressure

$p' = p_e$ when $p_e > p$

$p' = p_c$ when $p_e < p_c$

$$\frac{p_c}{p} = \left(\frac{2}{m+1}\right)^{\frac{m}{m-1}}$$ (the 'critical pressure ratio' for flow through the exhaust valve)

The Pressure Balance equation

The next stage of the analysis is to calculate the change in cylinder pressure following a small movement of the piston, thus leading to a differential equation which can be integrated to yield the pressure - volume diagram for a complete cycle. The increase in volume behind the piston is filled partly by the steam flowing in through the inlet port, and partly by the expansion of the steam that is already in the cylinder. We have already calculated the former quantity. The latter can be obtained by the following argument. Suppose the volume of steam in the cylinder changes by an amount δV and the pressure by an amount δp; the steam already in the cylinder will expand by an amount $m(\partial v/\partial p)_s \,\mathbf{.}\, \delta p$ where m is the mass of steam in the cylinder, and the partial derivative of v is at constant entropy, s. Thus the volume of new steam required is given by:

$$\delta V - m(\partial v/\partial p)_s . \delta p = \delta V - (V/v)\,(\partial v/\partial p)_s . \delta p$$

For a reversible polytropic expansion with an expansion index *n*, $(\partial v/\partial p)_s = (1/n)(v/p)$. Using this and substituting the inflow of steam from Equation (6), we get the pressure balance equation for an interval of time δt (corresponding to δp and δv) in the form:

$$C_c \boldsymbol{.} A_i \boldsymbol{.} \sqrt{p_o v_o}\, \frac{p_o}{p} \sqrt{2\frac{n}{n-1}\left\{\left(\frac{p'}{p_o}\right)^{\frac{2}{n}} - \left(\frac{p'}{p_o}\right)^{\frac{n+1}{n}}\right\}} \boldsymbol{.}\, dt \quad = \quad dV - \frac{V}{np}dp \qquad \text{..........} \;(10)$$

Rearranging and changing the variables to φ ($= p/p_o$) and crankangle θ, and writing the angular velocity $\omega = d\theta/dt$, and $\phi' = p'/p_o$

$$\frac{d\phi}{d\theta} = \left[C_d A_i \sqrt{p_o v_o}\,\boldsymbol{.}\frac{1}{\omega\phi V}\sqrt{\frac{2}{n-1}\left\{(\phi')^{\frac{2}{n}} - (\phi')^{\frac{n+1}{n}}\right\}} - \frac{1}{V}\frac{dV}{d\theta}\right] n\phi \qquad \text{.......................} \;(11)$$

The quantities $(dV/d\theta)$ and V can be evaluated from Equation 1, from which

$$\frac{V}{A_p s} = \frac{V_{cv}}{A_p s} + \left[c/s + 0.5 - \sqrt{(c/s)^2 + (\mathbf{sin}\,\theta/2)^2} - \mathbf{cos}\,\theta/2\right] = \xi_{cv} + \xi$$

$$\frac{1}{A_p s}\frac{dV}{d\theta} = \left[\frac{\mathbf{sin}\,\theta\boldsymbol{.}\mathbf{cos}\,\theta}{4\sqrt{(c/s)^2 + (\mathbf{sin}\,\theta/2)^2}} + \mathbf{sin}\,\theta/2\right] = \frac{d\xi}{d\theta}$$

where V_{cv} = clearance volume, A_p = piston area, and $\xi_{cv} = V_{cv}/A_p s$

The flow area through the valve A_i can be evaluated from the port opening (Equation (4) for inlet and Equation (5) for exhaust) and the perimeter of the ports, making due allowance for bridging.

A similar analysis can be carried out for the exhaust phase. The final version of the two forms of the differential equation are given below.

Admission

$$\frac{d\phi}{d\theta} = \frac{n\phi}{\xi_{cv} + \xi}\left\{G\boldsymbol{.}\varsigma_i \frac{1}{\phi}\sqrt{\frac{2}{n-1}\left[(\phi')^{2/n} - (\phi')^{n+1/n}\right]} - \frac{d\xi}{d\theta}\right\} \qquad \text{.............................} \;(12)$$

where the dimensionless port opening, $\varsigma_i = \frac{y_i}{L} = \frac{r_{eq}}{L}\boldsymbol{.}\mathbf{sin}(\theta + \psi) - 1$

and $\quad \phi' = \phi \quad$ when $\phi \geq \left(2/n+1\right)^{n/n-1}$ (i.e. subsonic flow through port opening)

$\phi' = \left(2/n+1\right)^{n/n-1}$ when $\phi \leq \left(2/n+1\right)^{n/n-1}$ (i.e sonic flow through port opening)

Exhaust

$$\frac{d\phi}{d\theta} = \frac{-m\phi}{\xi_{cv} + \xi}\left\{G\boldsymbol{.}\varsigma_o \phi^{n-1/2n}\sqrt{\frac{2m}{n(m-1)}\left[(\phi'')^{2/m} - (\phi'')^{m+1/m}\right]} + \frac{d\xi}{d\theta}\right\} \qquad \text{......................} \;(13)$$

where the dimensionless port opening, $\varsigma_o = \frac{y_o}{L} = \frac{r_{eq}}{L}\boldsymbol{.}\mathbf{sin}(\theta + \phi) - \frac{L_{ex}}{L}$

and $\phi'' = \phi_e/\phi$ when $\phi_e/\phi \geq \left(2/m+1\right)^{m/m-1}$ (i.e. subsonic flow through port opening)

$\phi'' = \left(2/m+1\right)^{m/m-1}$ when $\phi_e/\phi \leq \left(2/m+1\right)^{m/m-1}$ (i.e sonic flow through port opening)

In Equations (12) and (13) $G = \frac{C_d}{\omega}\frac{(valve\_perimeter).L}{A_p s}\sqrt{np_o v_o}$ . . . . (14)

and $\phi_e = p_e / p_o$

Equation (12), as well as serving the admission phase, also serves the expansion phase because the valve closes at cut-off, thus making y_i zero. The equation will then be found to reduce to the statement that $dp/dV = -np/V$, which integrates to the expansion law pv^n.=*constant.* Similarly, Equation (13) serves the compression phase as well as the exhaust phase.

Equations (12) and (13) represent a first order ordinary differential equation defining the dimensionless pressure ϕ as a function of crank angle θ. In the accompanying computer programme (written in C and compiled for a PC) this equation is integrated using a Runge-Kutta routine in which the step length is automatically varied to maintain a specified accuracy. The initial pressure at the start of the stroke is unknown because even when the inlet port is open the cylinder pressure may not rapidly come into equilibrium with the steam chest pressure. The procedure adopted is to start the first integration around the cycle by assuming that the cylinder pressure is equal to the steam chest pressure; completion of this cycle then provides a suitable initial condition for the second integration.

The process as described above has assumed implicitly that the exhaust pressure is known. In reality there will be a pressure drop through the "front end", and the magnitude of this drop will change with the rate of flow of steam through the blastpipe. This effect can be simulated by repeating the integration beyond the two cycles described above, calculating the total steam flow and the blastpipe pressure drop after each integration and using this to determine the exhaust pressure for the next integration. This is then repeated until there is no significant change in the diagram. The method of calculating the blastpipe pressure drop is described in the Appendix.

The RK routine is capable of integrating several first order differential equations simultaneously, providing the opportunity to integrate the pressure and also the flow through the inlet valve. The former leads directly to the Mean Effective Pressure for the cycle, and the latter to the Steam Consumption.

APPENDIX 2 : BLASTPIPE PRESSURE

The main programme calculates the steam flow through the cylinders, so the mass flow rate through the blastpipe, *M*, is known. In order to determine the pressure ratio across the blast nozzle (of cross sectional area A_b) we can use a relationship such as Equation 6 which was used on the ports. Denoting the blastpipe pressure as p_e, and the atmospheric pressure as p_a we can write the relationship in the form:

$$\frac{M}{A_b} = p_a\sqrt{\frac{2n}{n-1}}\frac{1}{\sqrt{p_e v_e}}\sqrt{\left(\frac{p_e}{p_a}\right)^{\frac{2(n-1)}{n}} - \left(\frac{p_e}{p_a}\right)^{\frac{n-1}{n}}} \quad . . . . (15)$$

We wish to solve this in order to establish a relationship between M/A_b and the ratio p_e / p_a but unfortunately the quantity $p_e v_e$ is not known. The way out is to recognise that over the pressure range 0.1 to 0.2 MPa it can, to a good approximation, be represented as a function of the exhaust enthalpy h_e ; this in turn can be evaluated from the initial enthalpy and the work done in the cylinders (First Law analysis). Thus:

$$f(h_e) = p_e v_e = -550.6 + 0.298h_e - 0.0000106h_e^2 \text{ kJ/kg} \quad . . . . (16)$$

With $\sqrt{p_e v_e}$ known, Equation 15 can be recognised as a binomial in $\left(p_e/p_a\right)^{n/n-1}$ and solved as:

$$\frac{p_e}{p_a} = \left[\frac{1}{2} + \sqrt{\frac{1}{4} + \beta^2} \right]^{n/n-1} \qquad \text{. . . . . (17)}$$

where $\beta = \frac{M}{A_b} \frac{\sqrt{f(h_e)}}{p_a} \sqrt{\frac{n-1}{2n}}$

If the pressure ratio exceeds a critical value the flow through the blastpipe will reach sonic speed; beyond this ratio the mass flow will depend only upon the upstream pressure p_e .The critical pressure ratio in terms of the upstream pressure p_e and the throat pressure in the blast nozzle p_t is given by:

$$\frac{p_t}{p_e} = \left(\frac{2}{n+1} \right)^{n/n-1}$$

If this is inserted in Equation (6) in place of p'/p_o we obtain the following relationship:

$$p_e = \frac{M}{C_b A_b} \Bigg/ \sqrt{\frac{n}{f(h_e)} \cdot \left(\frac{2}{n+1} \right)^{\frac{n+1}{n-1}}} \qquad \text{. . . . . (18)}$$

Depending upon whether the pressure ratio across the nozzle exceeds the critical ratio for sonic flow, Equation 17 or Equation 18 is used in the numerical solution to determine the backpressure suffered by the cylinders. Equation (17) or Equation (18) can now be used in conjunction with the pressure balance equation for the exhaust phase, Equation (13), since the pressure p_e at inlet to the blastpipe may be taken to be the same as the exhaust pressure seen by the cylinders. Iteration is required since the exhaust pressure is initially taken to be atmospheric. The first pass through the pressure balance equation yields the mass flow m and the exhaust enthalpy h_e which enable the exhaust pressure to be revised using Equation (17) or Equation (18). This revised value is then used in the second pass through the pressure balance equation, and this process is repeated until the exhaust pressure stabilises.